Proceedings of the
International Conference on Mechanical Engineering 2011
(ICME2011) 18-20 December 2011, Dhaka, Bangladesh

ICME 11-TH-011

MIXED CONVECTION LAMINAR FLOW ALONG A HORIZONTAL PLATE SUBJECT TO STREAMWISE SINUSOIDAL SURFACE TEMPERATURE

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ABSTRACT

Mixed convection of a two-dimensional laminar incompressible flow along a horizontal flat plate with streamwise sinusoidal surface temperature has been numerically investigated for different values of Rayleigh number and Reynolds number for constant values of Prandtl number, amplitude and frequency of periodic temperature. The numerical scheme is based on the finite element method adapted to rectangular non-uniform mesh elements by a non-linear parametric solution algorithm. The fluid considered in this study is air. The results are obtained for the Rayleigh number and Reynolds number ranging from 10^2 to 10^4 and 1 to 100, respectively, with constant physical properties for the fluid medium considered. Velocity and temperature profiles, streamlines, isotherms, and average Nusselt numbers are presented to observe the effect of the investigating parameters on fluid flow and heat transfer characteristics. The present results show that the convective phenomena are greatly influenced by the variation of Rayleigh numbers and Reynolds number.

Keywords: Laminar Mixed Convection, Horizontal Flat Plate, Sinusoidal Surface Temperature.

1. INTRODUCTION

Mixed convection flows are important as they are found in many practical situations in nature and man-made devices. The process of heat transfer by mixed convection flow over horizontal surfaces occurs in many industrial and technical applications which include cooling of electronic devices, furnaces, lubrication technologies, chemical processing equipment, drying technologies, solar collector and heat exchangers. Flow over a flat horizontal plate is one of the most fundamental and classic heat transfer problem and is associated with countless heat transfer incidents. Majority of the previous studies of mixed convection over a horizontal flat plate is either considered for constant surface temperature or constant heat flux. Cases like mixed convection over horizontal flat plate for periodic temperature at the surface have not received much attention. Some relevant investigations on mixed convection and periodic boundary conditions have been carried out previously [1–5]. The purpose of the present study is to determine a through analysis of the flow and heat transfer for mixed convection over a horizontal flat plate for ranges of Rayleigh number (Ra) Reynolds number (Re) with constant Prandtl number (Pr), amplitude (ΔT) and the frequency (f_T) of the periodic temperature.

2. PROBLEM FORMULATION

In the present investigation, mixed convection a horizontal flat plate with sinusoidally varying surface temperature has been considered. To avoid the leading edge effect, a small section of the leading edge from the left has been considered as adiabatic and the periodic surface temperature has been considered after the adiabatic section. The thermal behavior and flow pattern over the plate has been studied for various Rayleigh number, Ra and Reynolds number, Re for constant Prandtl number, Pr; frequency of the periodic surface temperature, f_T. The left side of the computational domain has been considered as the inlet. The fluid media that has been considered to flow over the plate is air (Pr = 0.72). The schematic diagram of the investigate enclosure has been shown in the fig. 1. The Rayleigh number, Ra range for the investigation has been taken from 10^2 to 10^4. The Reynolds number, Re range for the investigation has been taken from 1 to 100. The frequency of periodic surface temperature, f_T ranges for the investigation has been taken as 1

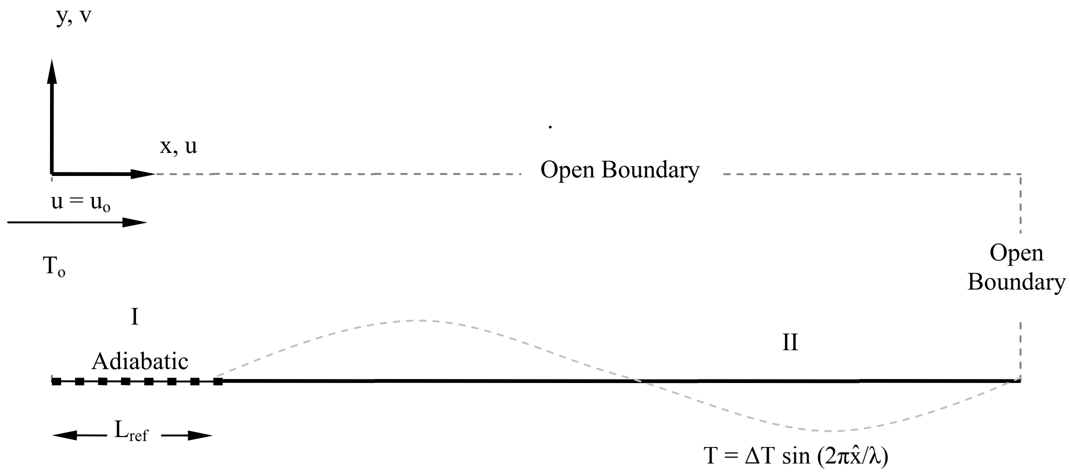

Fig 1. Physical model for the computational domain

3. MATHEMATICAL FORMULATION

3.1 Governing Equation

For this investigation, it has been assumed, the surface temperature exhibits small amplitude oscillation in $\hat{x}$ (the distance along the surface from beyond the leading adiabatic section) about a non-zero mean temperature. Mixed convection is governed by the differential equations expressing conservation of mass, momentum and energy. All the considered models are a horizontal flat plate having sinusoidally varying periodic surface temperature. The non-dimensional amplitude of the periodic temperature function is ΔT. The viscous dissipation term in the energy equation has been neglected. The Boussinesq approximation is invoked for the fluid properties to relate density changes to temperature change and to couple in this way the temperature field to the flow field. That is, the fluid properties are assumed to be constant except for the density which is considered to very linearly with temperature according to the Boussinesq approximation. Then the governing equations for steady mixed convection can be expressed in the dimensionless form as follows:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad (1)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \quad (2)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\frac{Ra\theta}{Re^2 Pr} \quad (3)$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{RePr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \quad (4)$$

3.2 Normalization

To obtain these above normalized equations (1), (2), (3) and (4) into non-dimensional form, the following parameters are defined.

$$Ra=\frac{g\beta L_{ref}^3\Delta T}{\eta\alpha},\ Re=\frac{u_o L_{ref}}{\eta}\ \text{and}\ Pr=\frac{\eta}{\alpha};$$
$$\text{where}\ \eta=\frac{\mu}{\rho}\ \text{and}\ \alpha=\frac{k}{\rho C_p} \quad (5)$$

$$X=\frac{x}{L_{ref}},\ Y=\frac{y}{L_{ref}},\ U=\frac{u}{u_o},\ V=\frac{v}{u_o},$$
$$P=\frac{p}{\rho u_o^2},\ \theta=\frac{T-T_o}{\Delta T},\ \Delta T=T_{pk}-T_{av} \quad (6)$$

Here, u_o is the velocity of the fluid media at inlet and C_p is the specific heat capacity at constant pressure which already has been mentioned before. The non-dimensional numbers seen in the above, Ra, Re and Pr are the Rayleigh number, Reynolds number and Prandtl number, respectively. L_{ref} is the characteristic length, α, β, θ, ρ, η, g, P, p, T are thermal diffusivity, thermal expansion coefficient, dimensionless temperature, fluid density, kinematic viscosity, gravitational acceleration, non-dimensional and dimensional pressure and dimensional temperature, respectively. U and V are the velocity components in the X- and Y-directions, respectively, θ is the temperature and P is the pressure. The average Nusselt number is defined as follows:

$$Nu=-\int_0^{L_{ref}}\frac{h.x}{k}dx=-\int_0^1\frac{x}{L_{ref}}\left.\frac{\partial\theta_s(X)}{\partial Y}\right|_{Y=0,1}dX$$
$$=-\int_0^1 X\left.\frac{\partial\theta_s(X)}{\partial Y}\right|_{Y=0,1}dX \quad (7)$$

Where, $\theta_s(X)$ and k are the dimensionless local temperature at the heated surface and thermal conductivity, respectively

3.3 Boundary Condition

For the Navier-stokes equation, the boundary conditions are no slip at horizontal flat plate and for the energy equation; it has been maintained at a periodic temperature distribution. To avoid leading edge effect, some portion from the leading edge of the horizontal flat plate has been considered adiabatic. The fluid considered for this study enters the computational domain from left. The boundary conditions and the flow domains are illustrated in Fig. 1. Mathematically those can be written as non-dimensional form.

Bottom surface (I)	$U = 0;\ V = 0;\ \dfrac{\partial\theta}{\partial Y} = 0$	(8)(a)
Bottom surface (II)	$U = 0;\ V = 0;\ \theta = \Delta\theta\sin(2\pi\hat{x}/\lambda)$	(8)(b)
Left surface	$U = 1;\ V = 0;\ \theta = 0$	(8)(c)
Right surface	$P = 0;\ \theta = \theta$	(8)(d)
Top surface	$P = 0;\ \theta = 0$	(8)(e)

4. NUMERICAL MODELING

4.1 Computational Procedure

The set of partial differential equations (1)-(4) with boundary conditions (8) are highly non-linear and coupled hence, difficult to solve them analytically. So, the momentum and energy balance equations have been solved by using the Galerkin weighted residual finite element technique. The continuity equation has been used as a constraint due to mass conservation. The basic unknowns for the above differential equations are the velocity components (U, V), the temperature θ and the pressure P. The computational domain has been discretized with linearly varying non-uniform rectangular mesh elements. The finite element (FE) model is implemented with two types of triangular Lagrange element: an element with linear velocity and pressure interpolations for the continuity and momentum equations and an element with a quadratic basis velocity and temperature interpolations for the energy equation. A stationary non-linear solver is used together with Direct (UMFPACK) linear system solver. These non-linear equations are solved iteratively using Broyden's method with a LU-decomposition pre-conditioner, always starting from a solution for a nearby Rayleigh number. The convergence of solutions is assumed when the relative error for each variable between consecutive iterations is recorded below the convergence criterion δ such that

$$\left| \frac{\Gamma^{n+1} - \Gamma^{n}}{\Gamma^{n+1}} \right| < \delta \qquad (14)$$

Where n is the Newton's iteration index and $\Gamma = U, V, P$ and θ. The convergence criterion is set to be 10^{-6}.

4.2 Grid Refinement Check

The distribution of mesh has been shown for Ra = 10^4, Ra = 5, f_T = 1, ΔT = 1, in Tables 1. The whole computational domain has been discretized with non-uniform rectangular mesh elements where the density of mesh element increases linearly along vertically downward direction. The discretization along horizontal direction initially increases linearly from the leading edge from left, at the adiabatic section. Along the periodically heated region, the discretization is uniform along x direction.

Five different meshes for each case, i.e. 150 × 30, 160 × 35, 170 × 40, 180 × 45 and 190 × 50 have been tested for this purpose.

Table 1: Grid distribution and sensitivity test

Grid (H × L)	Horizontal Direction		Vertical Direction	Average Nusselt
	Adiabatic Section	Heated Section		
150 × 30	15	135	30	4.235862
160 × 35	18	142	35	4.227756
170 × 40	20	150	40	4.222426
180 × 45	22	158	45	4.218588
190 × 50	25	165	50	4.215567

5. RESULT AND DISCUSSION

The working is air with Prandtl number, Pr = 0.7. The considered value of Rayleigh number, Ra, is 10^2, 2.5×10^2, 5×10^2, 7.5×10^2, 10^3, 2.5×10^3, 5×10^3, 7.2×10^3, and 10^4 and for Reynolds number, Re, is 1, 5, 10, 15, 20, 30, 40, 50, and 100. To observe the effect of Rayleigh number, Ra; Re, Pr, f_T and ΔT has been considered 5, 0.72, 1 and 1, respectively. Similarly, to observe the effect of Reynolds number, Re; Ra, Pr, f_T and ΔT has been considered 10^3, 0.72, 1 and 1, respectively. Results show that the aforementioned parameters have significant effects on the fluid flow and heat transfer for the cases considered.

5.1 Effect of Rayleigh Number

The thermal field and development of flow over a horizontal flat plate, having sinusoidally varying surface temperature, for Rayleigh number, Ra ranging from 10^2 to 10^4 is presented in Fig. 2, for Reynolds Number, Re = 5; the frequency of periodic temperature, f_T = 1; Prandtl number, Pr = 0.72 and the amplitude of periodic temperature, ΔT = 1. From isotherms (left column) and streamlines (right column), Fig. 2, It is clear that the thermal and flow fields are significantly effected by the variation of Rayleigh number, Ra. For all the other investigating parameters kept unchanged, isotherms have a dome like shape for lower Rayleigh number like Ra = 10^2, Fig. 2(a). The isotherms over the colder section of periodic surface temperature have more flat profile compared to the isotherms over the hotter section. As Rayleigh number increases, the isotherm over the hotter section transforms into a more tower like shape while the isotherm over the colder section becomes more and more flat with the increase of Rayleigh number. These flattened isotherms over the colder section push the isotherm over the hotter section to left side. For this reason, the tower like isotherms over the hotter section seems to be a bit leaned to the left for higher Rayleigh number like Ra = 10^4, Fig. 2(e). This change of shape over the hotter region indicates the increased buoyancy effect as Rayleigh number increases.

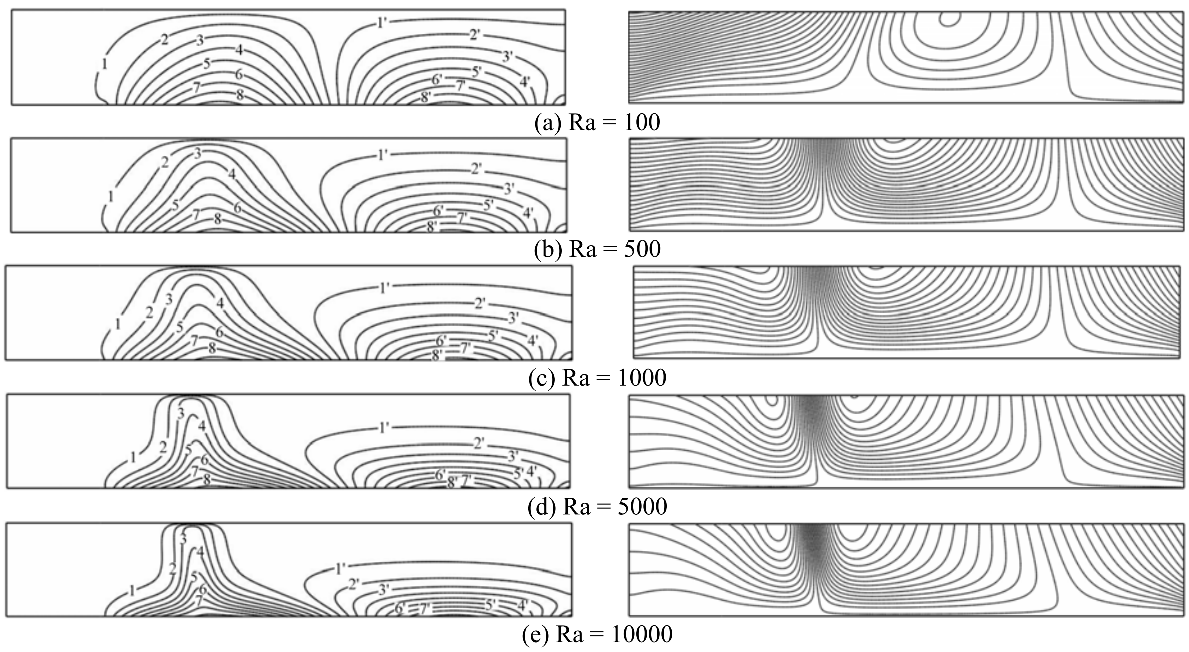

Fig 2. Isotherms (left column) and streamlines (right column) for different Rayleigh number, Ra at Re = 5, f_T = 1, Pr = 0.72, ΔT = 1

From the streamline plots for various Rayleigh number, Fig. 2, it is seen that the fluid media enters the domain and gradually goes vertically upward as it passes through the hotter periodic region due to buoyancy. Meanwhile, colder fluid media enters the domain from top where the colder region of periodic temperature resides. Some portion of the colder fluid media leaves the domain downstream. But, rest of the entered fluid media moves toward the hotter region as the fluid density is less there. Colder fluid entering the hotter region gets hot and leaves the domain going vertically upward. For lower Rayleigh number like Ra = 10^2, Fig. 2(a), the vertically upward and downward motion of the fluid media is more gentle compared to higher Rayleigh number. As Rayleigh number increases, the vertically upward and downward motion of fluid media becomes more sharp and steeper and the flow strength also increases. This indicates why the dome like isotherms changes their shape into tower like shape. While the flow strength increases with increase of Rayleigh number, the fluid enters the domain more abruptly and the region where the fluid media leaves the domain vertically upward shifts to left, like Rayleigh number, Ra = 10^4, Fig. 2(e). This justifies the leftward leaning tendency of isotherms for higher Rayleigh number.

Fig. 3 shows, the heat transfer characteristic is also affected significantly with the variation of Rayleigh number. As Rayleigh number increases, the magnitude of the characteristic parameter for heat transfer, average Nusselt number, also increases in a non linear manner. The trend of change of the magnitude of average Nusselt number changes with the change of Reynolds number.

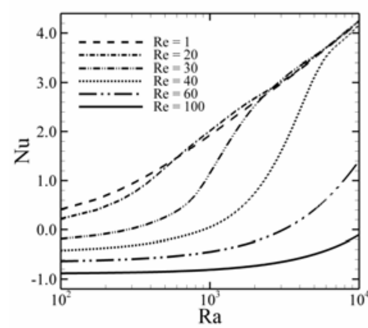

Fig 3. Average Nusselt number, Nu versus Rayleigh number, Ra for various Re at Pr = 0.72, f_T = 1, ΔT = 1

5.2 Effect of Reynolds Number

The thermal field and development of flow over a horizontal flat plate, having sinusoidally varying surface temperature, for Reynolds number, Re ranging from 1 to 100 is presented in Fig. 4 for Rayleigh Number, Ra = 10^3; the frequency of periodic temperature, f_T = 1; Prandtl number, Pr = 0.72 and the amplitude of periodic temperature, ΔT = 1. The isotherms (left column) and streamlines (right column), Fig. 4, clearly show that there is a significant impact of the variation of Reynolds number, Re on the thermal and flow fields. For all the other investigating parameters kept unchanged, isotherms have a dome like shape for lower Reynolds number like Re = 1, Fig. 4(a) and the isotherms over the colder section of periodic surface temperature have more flat profile compared to the isotherms over the hotter section, which is similar to the isotherms for lower Rayleigh number. As Reynolds number increases, the isotherm over both the hotter and the colder section gradually begin to transform into more like a shark fin shape, Fig. 4 (c), (d), (e). From Reynolds number, Re = 30, the isotherm originated from the hotter region of the flat plate tend to move over the colder region and for Reynolds number, Re = 100, the hotter isotherms are well extended over the colder surface temperature region.

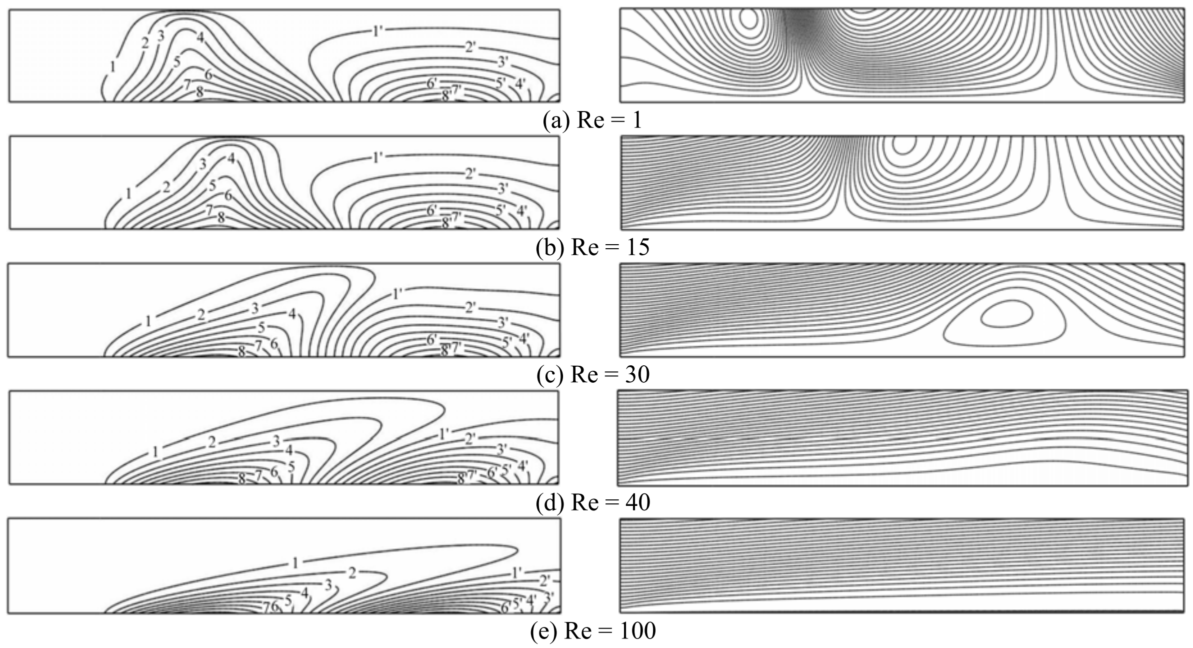

Fig 4. Isotherms (left column) and streamlines (right column) for different Reynolds number, Re at Ra = 10^3, f_T = 1 Pr = 0.72, ΔT = 1

Unlike the effect of lower Rayleigh numbers, for lower Reynolds number like Re = 1 at Fig. 4(a), the fluid media enters the domain abruptly. Then it gradually goes vertically upward as it passes through the hotter periodic region due to buoyancy. In the meantime, colder fluid media enters the domain from top where the colder region of periodic temperature resides and splits into two portions. One portion of the colder fluid media leaves the domain downstream and the other one moves toward the hotter region and leaves the domain going vertically upward. With increase of the Reynolds number, the vertically upward motion of the fluid seems to shift toward right side as the flow strength increases with the increase of Reynolds number. For Reynolds number, Re = 30, Fig. 4(c), the flow becomes strong enough for the colder fluid, that entered from the left side of the domain, to suppress the vertically upward motion and to flow over along the whole length of the plate. As a result, vortices of hotter fluid are originated over the hotter region of the plate. As Reynolds number increases more like Re = 40, Fig. 4(d), or above, those vortices disappear.

Fig. 5 illustrates the nature of the heat transfer characteristic with the variation of Reynolds number. For lower Reynolds numbers, the magnitude of average Nusselt number increases linearly with the increase of Reynolds number. As Reynolds number further increases, the value of average Nusselt number drastically falls down an exponential trend. This happens as the temperature gradient at vertical direction reduces along the vertical distance considered (see Fig. 4, left column) in the average Nusselt number calculation (see Eq. 7) as Reynolds number increases. As Rayleigh number changes, the trend remains similar.

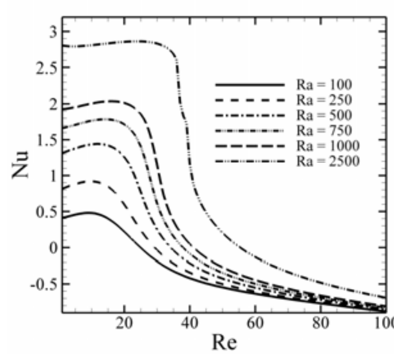

Fig 5. Average Nusselt number, Nu versus Reynolds number, Re for various Ra at Pr = 0.72, f_T = 1, ΔT = 1

6. CONCLUSION

A numerical study has been carried out on mixed convection over a horizontal flat plate with sinusoidally varying surface temperature for various parameters. The important outcomes of the present investigation are:

(1) For a convective fluid, for the model considered, having all other parameters kept constant, for the range of Rayleigh number, Ra from 10^2 to 10^4, the domination of convection over diffusion heat transfer as the value of Rayleigh number increases.

(2) For a convective fluid, for the model considered, having all other parameters kept constant, for the range of Rayleigh number, Ra from 10^2 to 10^4, the heat transfer seem to enhance as the value of Rayleigh number increases.

(3) For a convective fluid, for the model considered, having all other parameters kept constant, for the range of Reynolds number, Re from 1 to 100, the heat transfer seem to enhance at the beginning as the value of Reynolds number increases but the rate diminishes drastically with further increment of Reynolds number.

7. REFERENCES

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8. NOMENCLATURE

Symbol	Meaning	Unit
C_p	Specific heat capacity	J/Kg.K
f_T	frequency of periodic temperature, $[\hat{x}/\lambda]$	1/m
g	gravitational acceleration	m/s^2
h	convective heat transfer coefficient	W/m^2K
L	length along the plate	m
Nu	Nusselt number, Eq.(7)	
p	pressure	N/m^2
P	dimensionless pressure, $[p/\rho u_o^2]$	
Pr	Prandtl number, η/α	
Ra	Rayleigh Number, $[g\,\beta L^3_{ref}\Delta T/\eta\alpha]$	
Re	Reynolds Number, $[u\,L_{ref}/\eta]$	
T	temperature	K
ΔT	amplitude of periodic temperature	K
T_{av}	average temperature of the temperature function	K
T_o	temperature of the inlet air	K
T_{pk}	peak temperature of the temperature function	K
u	velocity component in x-direction	m/s
u_o	velocity at inlet	m/s
U	dimensionless velocity component in X-direction, $[u/u_o]$	
v	velocity component in y-direction	m/s
V	dimensionless velocity component in Y-direction, $[v/u_o]$	
$\hat{x}$	dimensionless length of heated region	
x, y	Cartesian co-ordinates	m
X, Y	dimensionless Cartesian co-ordinates, $[(x,y)/L_{ref}]$	
Greek Symbol		
Γ	dummy variable	
k	thermal conductivity of fluid	W/mk
α	thermal diffusivity	m^2/s
β	coefficient of volumetric expansion	1/K
θ	dimensionless temperature, $[T\text{-}T_o/\Delta T]$	
θ_s	local dimensionless surface temperature	
λ	wave length of the sinusoidal temperature function	m
μ	dynamic viscosity	$N\text{-}s/m^2$
η	kinematic viscosity	m^2/s
ρ	fluid density	kg/m^3
Subscript		
av	average	
o	inlet	
p	constant pressure	
pk	peak value	
ref	reference	
s	surface	
T	temperature	
w	wall	

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